Distinguishability of some Dark Energy Models

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Abstract

Dark energy hypothesis aims to explain the accelerated expansion of universe as indicated by evidence from Type Ia supernovae (SN 1a), Cosmic microwave background (CMB) and Baryon acoustic oscillations (BAO) data. Various models have been formulated to explain accelerated expansion. In this paper, we review the Quintessence and Barotropic fluid models. We will discuss various parameters that can be used to distinguish these models in w-w' plane, where w is dark energy equation of state (EOS) and w' is the derivative of w with respect to the logarithm of scale factor. We will also analyze the behavior of w'', that is the second derivative of w with respect to logarithm of scale fact, for phantom and non-phantom region.

1 Introduction

Evidence shows that universe is expanding at accelerated rate [[1]-[4]]. Observations indicate 70% of universe is composed of dark energy that has negative pressure and drives late time acceleration [5].We can explain negative pressure by considering universe as an elastic ball such that we are inside it. The area vector of the inner surface of ball is pointing towards the center where as the force applied on inner wall is radially outwards hence if we try to express this as an equation we will end up writing $\bar{\mathbf{F}}$=$-\bar{\mathbf{A}}$p, where $\bar{\mathbf{F}}$ is force vector, $\bar{\mathbf{A}}$ is area vector and p is pressure.

To explain the accelerated expansion, dark energy models have been classified into three broad categories namely Thawing, Freezing and Barotropic Fluid models. There is a possibility of other models that explain this phenomenon with greater clarity but till date these models are consistent with observational data at 95.4% confidence level.

The formulation of such models has led cosmologists to search for distinguishing parameters to justify experimental observations that fit these models. For this purpose, Caldwell and Linder [6] have examined these models in w-w' plane, where $w = \frac{p}{\rho}$ and $w' = \frac{dw}{d\ln(a)}$. We will now discuss these models in detail.

We have divided this paper into four main Sections. In Section 2 of this paper, we analyze Quintessence models and behavior of w and its derivatives. Section 3 deals with Barotropic Fluid model wherein we derive equation of w' in terms of w and speed of sound. We have also discussed phantom and non-phantom limits and corresponding behavior of equation of state. Section 4 concludes this paper with an overall analysis of the models discussed and w, w' and w'' behavior of each of them.

2 Quintessence Models

Quintessence model [[6]-[7]] involves a time varying scalar field and allows for the time evolution of equation of state. The domain of w in which this model is formulated is given by $w > -1$. A quintessence model is further classified into thawing and freezing model based on whether $w' > 0$ or $w' < 0$ respectively. We will take up each of these models separately and will try to analyze the bounds and physics of each of them.

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2.1 Thawing Model

As stated previously thawing model is formulated in domain $w > -1$ and $w' > 0$. The equation of state is at early times is $w \approx -1$. This can be seen from location of thawing model in Fig.(1). The limits for w' used for plotting this figure are $(1 + w) < w' < 3(1 + w)$ [7]. Since, $w' > 0$ for this model, this means, as one moves from past to present w becomes less negative indicating a decrease in $\frac{p}{\rho}$ ratio. Now, for adiabatic expansion, total energy must remain constant, hence with expansion, volume increases thus decreasing ρ, the energy density. Since, ρ decreases with expansion p must also decrease. Decrease in p must lead to the decrease in expansion rate. Here, decrease in expansion rate means w' becoming more positive because a negative w indicates expansion. This indicates $w'' > 0$ for thawing model. We can easily verify this analysis from Fig.(1) where the limits of w' become more positive as w changes from -1 to 0.

2.2 Freezing Model

This model is formulated for limits $w > -1$ and $w' < 0$ as can be seen from Fig.(1). The limits for w' used for plotting this figure are $3w(1 + w) < w' < 0.2w(1 + w)$ [7]. For freeing model, equation of state is taken to be $w > -1$ and $w' < 0$ initially and the field is said to be frozen at late times to $w = -1$ and $w' \to 0$. According to this model, as we move from past to present, the pressure responsible for expansion of universe increases *i.e* becomes more negative. This can be easily verified by the fact that $w' < 0$. Moreover, one can easily see from Fig.(1) that $w'' > 0$ for this model.

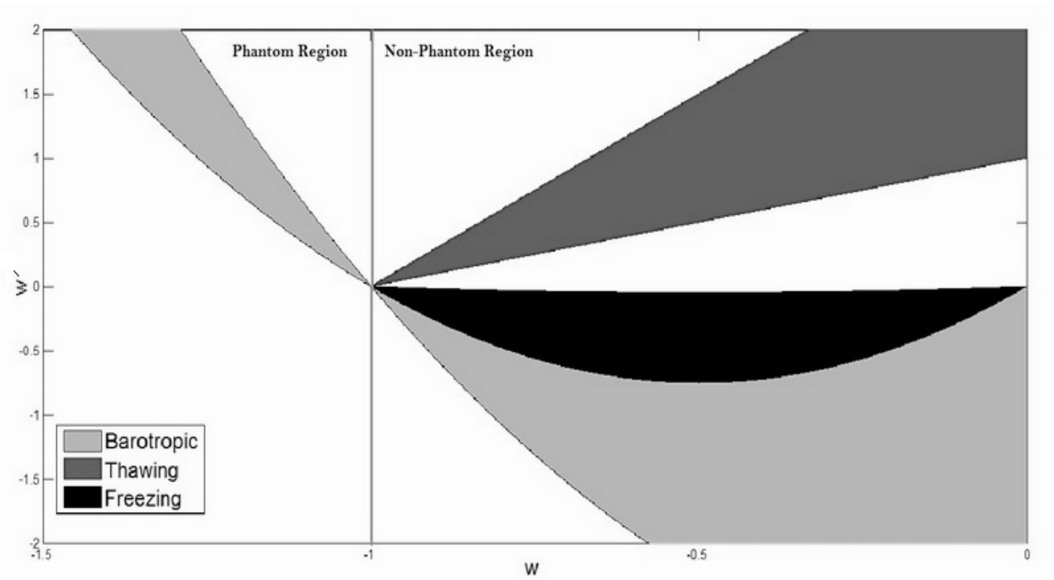

Figure 1: Combined plot of w-w' for three the models of dark energy. Left part of figure represents phantom limits with corresponding Barotropic model. Right part represents non-phantom limits with Barotropic and Quintessence Models.

3 Barotropic Fluid Model

Dark energy can be modeled as a barotropic fluid with a varying equation of state. The prototype for this sort of model is Chaplygin gas [[8],[9]]. For barotropic fluid model, the pressure is given as an explicit function of density and vice versa.

$$p = f(\rho) \tag{1}$$

Assuming that on macroscopic scale universe doesn't undergo phase transition. We can say that there is no exchange of heat if we consider universe as thermodynamic system. In other words, the expansion is adiabatic in nature. Hence, from first law of thermodynamics, we can write

$$dQ = dU + dW = 0 \implies dU = -PdV \tag{2}$$

Now, we define energy density ρ as ratio of internal energy U of the universe to its volume V

This gives us,

$$\begin{aligned}\rho = \frac{U}{V} &\implies d\rho = d\left(\frac{U}{V}\right) = \frac{1}{V}dU - \frac{U}{V^2}dV \\ &\implies d\rho = -(p+\rho)\frac{dV}{V}\end{aligned} \tag{3}$$

Assuming that volume of the universe is proportional to cube of scaling factor a^3,

$$\begin{aligned} & d\rho = -3(p+\rho)\frac{da}{a} \implies \frac{d\rho}{d\ln(a)} = -3(1+w)\rho \\ \text{Now,}\quad & w' = \frac{dw}{d\rho}\frac{d\rho}{d\ln(a)} \\ \text{Hence,}\quad & w = \frac{p}{\rho} \implies \frac{dw}{d\rho} = \frac{1}{\rho}\left(\frac{dp}{d\rho} - w\right) \\ & w' = \frac{dw}{d\ln(a)} = -3(1+w)\left(\frac{dp}{d\rho} - w\right) \\ \text{Now, } c_s \text{ is speed of sound then,}\quad & \frac{dp}{d\rho} = c_s^2 \end{aligned} \tag{4}$$

Hence, we have,

$$w' = \frac{dw}{d\ln(a)} = -3(1+w)\left(c_s^2 - w\right) \tag{5}$$

Using above derivation we can figure out limits of w' using arguments of causality and stability. To ensure causality, we must have $c_s^2 \leq 1$, which gives us the lower bound $w' \geq -3(1+w)(1-w)$. To ensure stability, we must have $c_s^2 \geq 0$, which gives us upper bound $w' \leq 3w(1+w)$. From Fig.(1) we can clearly see that barotropic fluid model lies both in phantom ($w < -1$) and non-phantom ($w > -1$) region. However, the causality and stability bounds are reversed as one goes from non phantom to phantom region.

3.1 Behaviour in Phantom and Non-Phantom Region

We tried to study behavior of barotropic fluid model in Phantom and Non-Phantom regions using w-z plots for given values of w'. Here, z is the redshift.

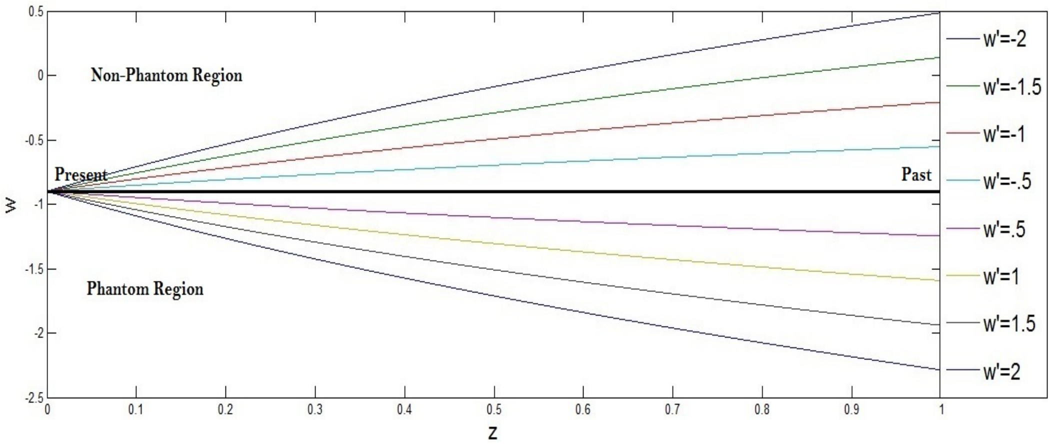

Figure 2: w vs. z plot of barotropic fluid model for various w' values

In Fig.(2), we can clearly see that for non-phantom region, as one moves from past to present, w becomes more negative, indicating a possible increase in pressure p or decrease in energy density ρ or both. The decrease in energy density can be explained if we consider total energy content of the universe to be constant due to adiabatic expansion of universe. Then with the increase in volume ρ should decrease. For phantom region w becomes less negative with decrease in z.

Furthermore, we have taken four parameterizations for c_s^2, fourth being a hybrid of first two.

$$\begin{aligned} &1) \quad c_s^2 = w^2 \\ &2) \quad c_s^2 = exp(w) \\ &3) \quad c_s^2 = w^{-2} \\ &4) \quad c_s^2 = \alpha * exp(w) + \beta * w^2, \text{Here } \alpha \text{ and } \beta \text{ are solved for limits } 0 \le c_s^2 \le 1 \end{aligned} \tag{6}$$

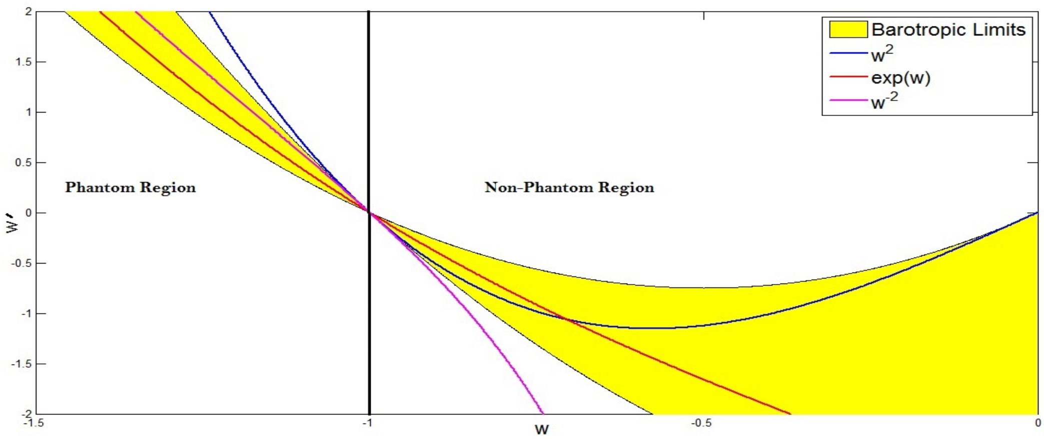

Figure 3: w vs. w' plot of barotropic fluid model for three parameterizations of c_s^2. Yellow region represents the limits of barotropic model and individual plots are made for various parameterizations of c_s^2

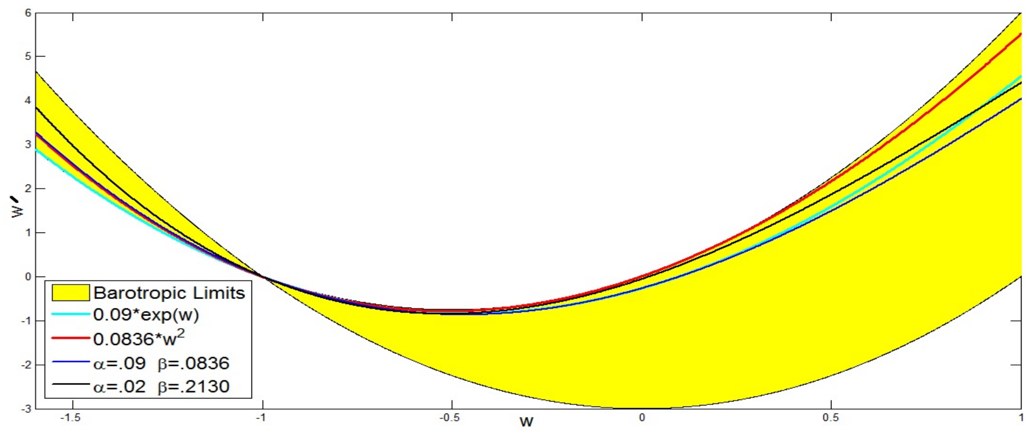

Figure 4: w vs. w' plot of barotropic fluid model for hybrid parameterization of c_s^2. Yellow region represents the limits of barotropic model and individual plots are made for various α and β values in hybrid parameterization.

We tried to test the validity of these parameterizations by substituting these in Eq.(5) and plotting corresponding w-w' plane. As can be seen from Fig.(3) that $exp(w)$ covers the range of interest of w within the limits of w' whereas, w^2 works well in non-phantom limits and w^{-2} in phantom limits. We further realized from Fig.(4) that the hybrid parameterizations are more effective in covering entire region of interest and hence are better approximations for c_s^2. We will use these parameterizations accordingly in both the regions for studying barotropic model.

In Fig.(5), for non-phantom limits, we can see that w becomes less negative with increase in value of z. That is, as we go to the past the $\frac{p}{\rho}$ ratio becomes less negative then its present value. On the contrary, for phantom limits, as we go to the past, value of w becomes more negative, which simply says that more pressure was required in the past for expansion as compared to present. One of the possible ways to distinguish phantom from non-phantom region of barotropic fluid model is to see the behavior of w' for exponential and hybrid plots in both the regions. In case of phantom region, the slope w' becomes less positive as redshift z decreases. Hence, w'' is negative. Whereas, in case of non-phantom region the slope becomes less negative with decrease in z. Hence, w''

is positive. Therefore, w'' vs z plots can effectively distinguish between these two regions in case of barotropic fluid models.

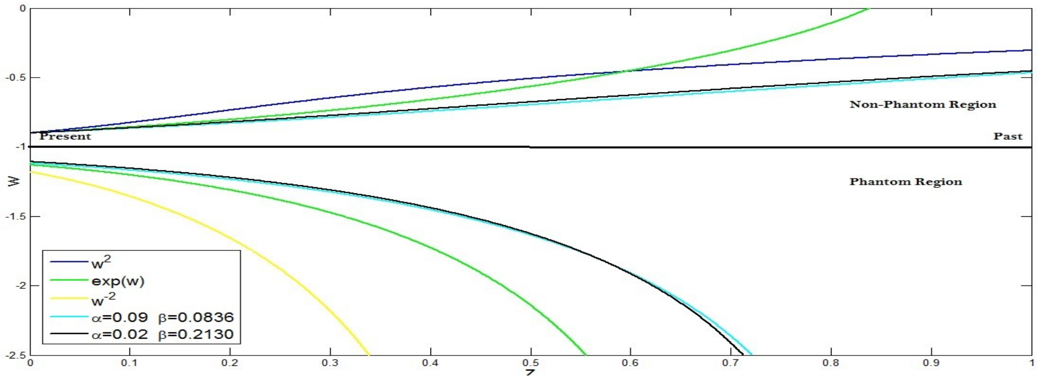

Figure 5: w vs. w' plot of barotropic fluid model for various parameterizations of c_s^2

We have summarized our discussion in last paragraph in Fig.(6). Here, thick lines show plots for phantom region whereas, thin lines show plots for non-phantom region.

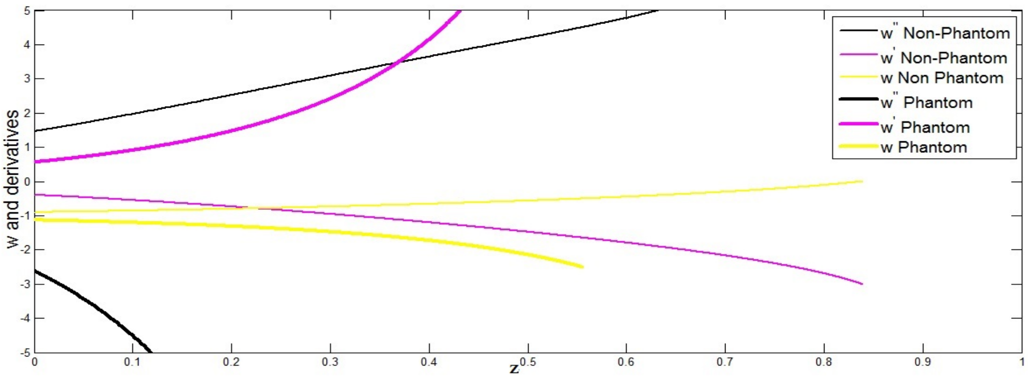

Figure 6: Plot of w and its derivatives vs. z for phantom and non-phantom region

4 Conclusions

In this paper, we have analyzed the Quintessence and Barotropic fluid models of Dark Energy. Analysis has shown that studying w'' behavior can actually distinguish phantom from non-phantom region. Table 1 summarizes the behavior of w, w' and w'' for each of the models discussed so far. Besides this, we also tried to figure out c_s^2 parameterization as functions of w that obey w' bounds within the region of interest. We found linear combinations of $exp(w)$ and w^2 are effective in parameterizing c_s^2 for a given region of interest. In our future work, we will further try to analyze thawing and freezing models and their w'' plots, to come up with a new w'' vs. z plane that efficiently distinguishes phantom and non-phantom limits.

Table 1: Phantom and Non- phantom behavior of w and its derivatives

Model	w	w'	w''
Phantom-Barotropic	< -1	> 0	< 0
Non Phantom-Barotropic	> -1	< 0	> 0
Thawing	> -1	> 0	> 0
Freezing	> -1	< 0	> 0

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