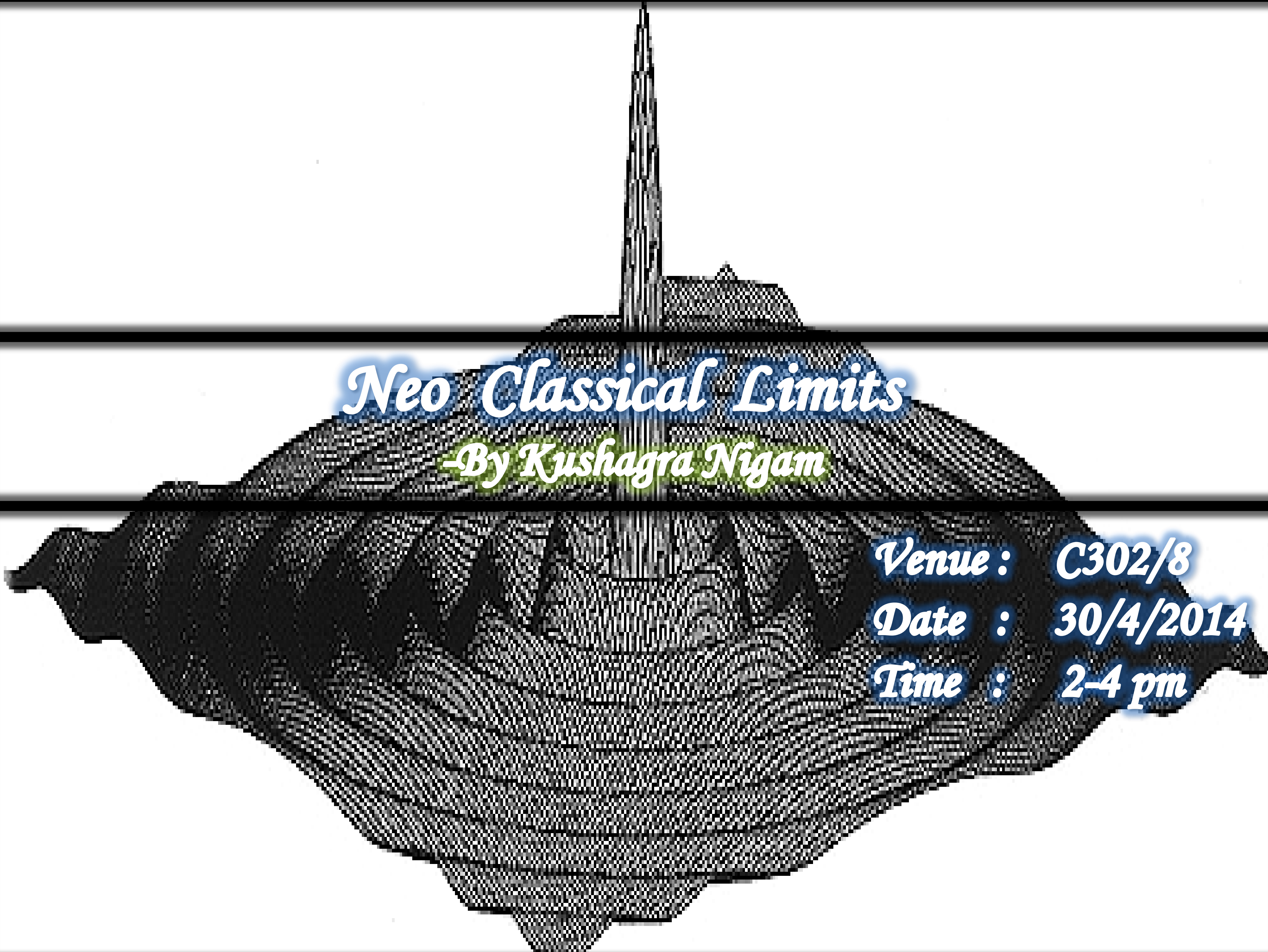


Neo Classical Limits

-By Kushagra Nigam

Venue : C302/8

Date : 30/4/2014

Time : 2-4 pm

Goals Of Seminar

- *To illustrate quantum harmonic oscillators.*
- *To understand coherent states and their classical behavior.*
- *To understand the classical limits of quantum systems.*

Quantum Harmonic Oscillator

$$\hat{H} = \frac{1}{2}k\hat{x}^2 + \frac{\hat{p}^2}{2m}$$

$$a^\dagger = \sqrt{\frac{mw}{2\hbar}}\left(\hat{x} - \frac{i}{mw}\hat{p}\right)$$

$$a = \sqrt{\frac{mw}{2\hbar}}\left(\hat{x} + \frac{i}{mw}\hat{p}\right)$$

$$a^\dagger a = N$$

$$\hat{x} = \sqrt{\frac{\hbar}{2mw}}(a^\dagger + a)$$

$$\hat{p} = \sqrt{\frac{\hbar m w}{2}}\ \frac{(a - a^\dagger)}{i}$$

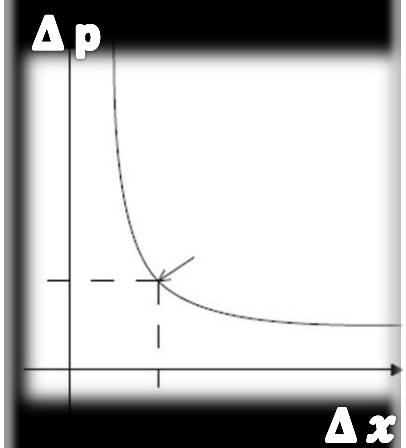
$$[a, a^\dagger] = 1$$

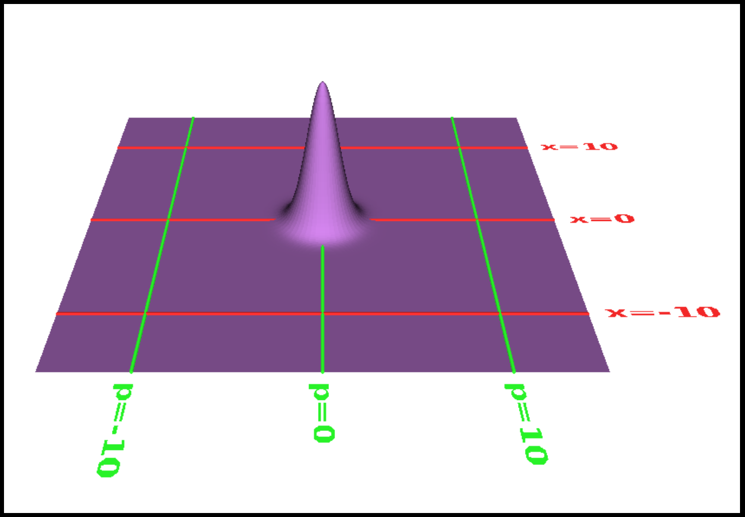
$$\Delta x = \sqrt{\left(n + \frac{1}{2}\right)\frac{\hbar}{mw}}$$

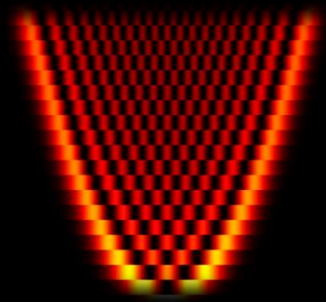
$$\Delta p = \sqrt{\left(n + \frac{1}{2}\right)\hbar m w}$$

Points to Note:

- *Stationary States - Mean and Variance are fixed*
- *Superposition is Non –stationary*
- *E and B fields exhibit same behavior as x and p in standing modes inside resonance cavity.*







Coherent States

$$\hat{H} = \frac{1}{2}k\hat{x}^2 + \frac{p^2}{2m}$$

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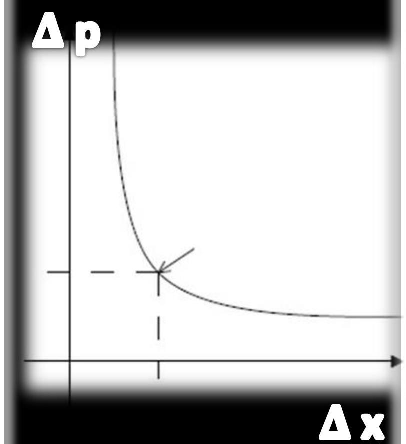
$$[a, a^\dagger] = 1$$

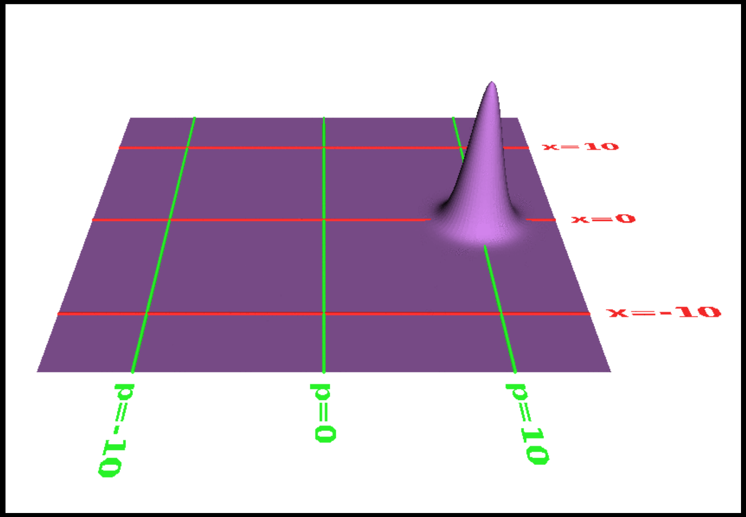
$$\Delta x = \sqrt{\frac{1}{2}\frac{\hbar}{mw}}$$

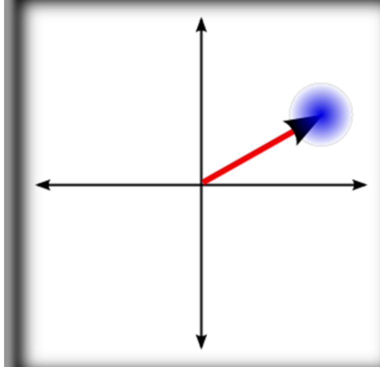
$$\Delta p = \sqrt{\frac{1}{2}\hbar mw}$$

Points to Note:

- *Overcompleteness and non-orthogonality.*
- *Mean oscillates with time.*
- *Classical reduction in limit $\hbar \rightarrow 0$.*
- *Laser itself is a coherent beam of bosons.*
- *Optical Laser – Boson is photon*
- *Atomic Laser – Boson is Atom.*







Quantum – Classical Correspondence

$$\hat{H} = \frac{1}{2}k\hat{x}^2 + \frac{p^2}{2m}$$

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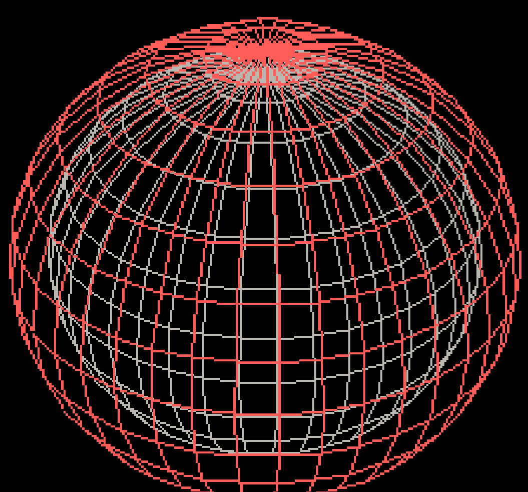
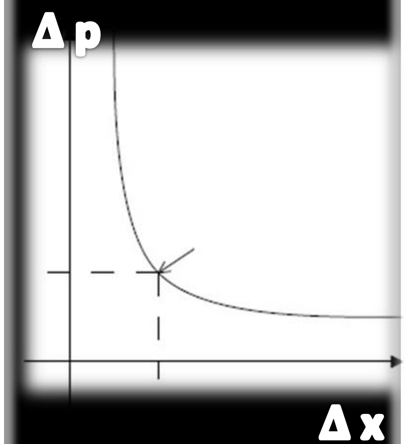
Transformations and Invariance:

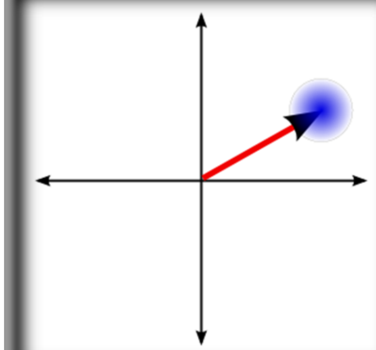
- *Definition of Transformation:*

- *Invariance of System under a Transform, meaning of symmetry:*

- *What is Invariant Here ??*







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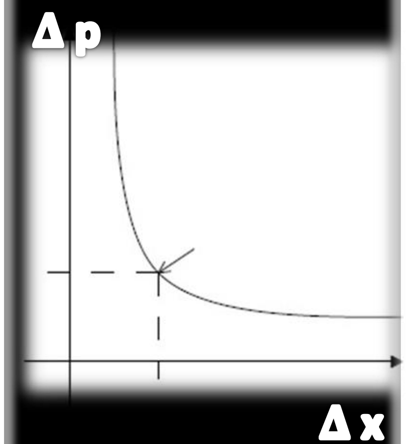
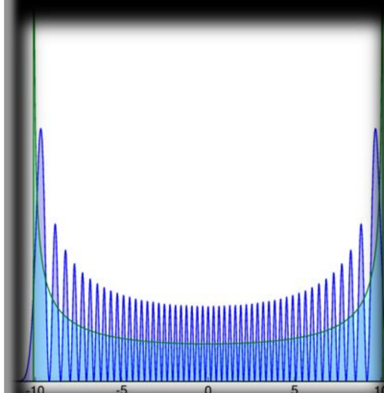
Vector Models:

- *Vector Space Invariance under Transformation:*
- *Invariance of Hamiltonian in Hilbert Space* H *under group of O(N) transformations:(N = dimensions):*

- *Commutation Relation Used:*
- *Formulation of Invariant Operators for O(N) invariant Hamiltonian:*
- *i)*
- *ii)*





Back to Quantum Oscillators

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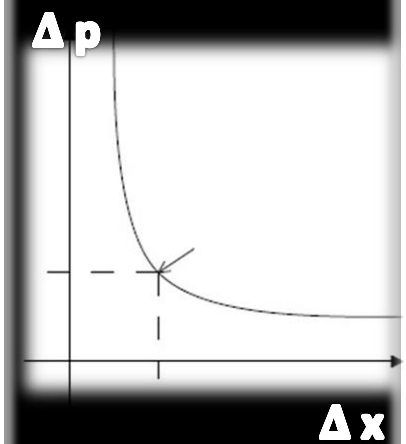
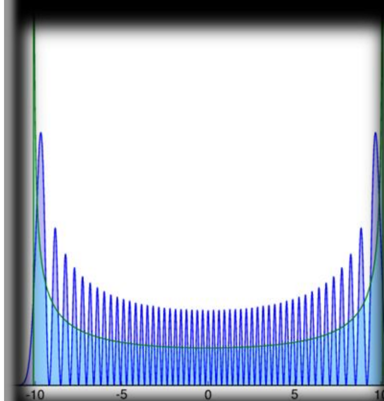
__*Consider our old Quantum Oscillator:*__

- *How to write Hamiltonian in terms of our invariant operators?*
- *What does the time evolution of these operators tell us about classical behavior?*

- *Commutation Relation Used:*
- *Formulation of Invariant Operators for O(N) invariant Hamiltonian:*
- *i)*
- *ii)*
- *iii)*





Black Board Mode

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Conclusion

Linking Theories is more tough than Creating them!!

Overcomplete Frames

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